QUANTUM SIZE EFFECTS IN CLASSICAL HADRODYNAMICS

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ABSTRACT

We discuss future directions in the development of classical hadrodynamics for extended nucleons, corresponding to nucleons of finite size interacting with massive meson fields. This new theory provides a natural covariant microscopic approach to relativistic nucleus-nucleus collisions that includes automatically spacetime nonlocality and retardation, nonequilibrium phenomena, interactions among all nucleons, and particle production. The present version of our theory includes only the neutral scalar (σ) and neutral vector (ω) meson fields. In the future, additional isovector pseudoscalar (π^+, π^-, π^0), isovector vector (ρ^+, ρ^-, ρ^0), and neutral pseudoscalar (η) meson fields should be incorporated. Quantum size effects should be included in the equations of motion by use of the spreading function of Moniz and Sharp, which generates an effective nucleon mass density smeared out over a Compton wavelength. However, unlike the situation in electrodynamics, the Compton wavelength of the nucleon is small compared to its radius, so that effects due to the intrinsic size of the nucleon dominate.

1. Introduction

During three previous winter workshops we have discussed a new microscopic many-body approach to relativistic nucleus-nucleus collisions based on classical hadrodynamics for extended nucleons, corresponding to nucleons of finite size interacting with massive meson fields.[1−3] The underlying foundations of this new theory, as well as applications to soft nucleon-nucleon collisions, have been published recently.[4,5] In this contribution, we would like to discuss future directions in the systematic development of this theory.

2. Extended Nucleon

We all know that the nucleon is a composite particle made up of three valence quarks plus additional sea quarks and gluons. When nucleons collide at very high energies, a few rare events correspond to the head-on or hard collisions between the individual quarks and/or gluons. For describing these events, the underlying quark-gluon structure of the nucleon is of crucial importance. Yet such hard collisions are extremely rare, typically one in a billion.[6] The vast majority of events correspond to soft collisions not involving individual quarks or gluons. For the description of

Fig. 1: Slice through the center of a nucleon. Although composed of the three indicated valence quarks plus sea quarks and gluons, for many purposes the nucleon can be regarded as a single extended object, with an exponentially decreasing mass density that is spherically symmetric in its instantaneous rest frame.

such events, an appealing idea is to regard the nucleon as a single extended object interacting with other nucleons through the conventional exchange of mesons (whose underlying quark-antiquark composition is ignored).

Experiments involving elastic electron scattering off protons have determined that the proton charge density is approximately exponential in shape,[7] with a root-mean-square radius of 0.862 ± 0.012 fm. Although many questions remain concerning the relationship between the proton charge density and the nucleon mass density,[8] it should be a fairly accurate approximation to regard them as equal. We therefore take the nucleon mass density to be

$$\rho(r) = \frac{\mu^3}{8\pi} \exp(-\mu r) \ , \tag{1}$$

with $\mu = \sqrt{12}/R_{\mathrm{rms}}$ and $R_{\mathrm{rms}} = 0.862$ fm. We show in fig. 1 a gray-scale plot of the mass density through the center of a nucleon calculated according to this exponential, with three valence quarks also indicated schematically.

3. Present Version

In the present version of our theory, we consider N extended, unexcited nucleons interacting with massive, neutral scalar (σ) and neutral vector (ω) meson fields. At

bombarding energies of many GeV per nucleon, the de Broglie wavelength of projectile nucleons is extremely small compared to all other length scales in the problem. Furthermore, the Compton wavelength of the nucleon is small compared to its radius, so that effects due to the intrinsic size of the nucleon dominate those due to quantum uncertainty. The classical approximation for nucleon trajectories should therefore be valid, provided that the effect of the finite nucleon size on the equations of motion is taken into account. The resulting classical relativistic many-body equations of motion can be written as[4,5]

$$M_i^* a_i^\mu = f_{\mathrm{s},i}^\mu + f_{\mathrm{v},i}^\mu + f_{\mathrm{s,ext},i}^\mu + f_{\mathrm{v,ext},i}^\mu \quad , \tag{2}$$

where M_i^* is the effective mass, $f_{\mathrm{s},i}^\mu$ is the scalar self-force, $f_{\mathrm{v},i}^\mu$ is the vector self-force, $f_{\mathrm{s,ext},i}^\mu$ is the scalar external force, and $f_{\mathrm{v,ext},i}^\mu$ is the vector external force.

These classical relativistic many-body equations of motion can be solved numerically without further approximation. In particular, there is

- No mean-field approximation
- No perturbative expansion in coupling strength
- No superposition of two-body collisions

We have thus far solved these equations for soft nucleon-nucleon collisions at $p_{\mathrm{lab}} =$ 14.6, 30, 60, 100, and 200 GeV/c to yield such physically observable quantities as scattering angle, transverse energy, radiated energy, and rapidity.[5] We found that the theory provides a physically reasonable description of gross features associated with the soft reactions that dominate nucleon-nucleon collisions. In addition, the present version of the theory permits a qualitative discussion of several important physical points:

- Effect of finite nucleon size on equations of motion
- Inherent spacetime nonlocality
- Particle production through massive bremsstrahlung

4. Additional Meson Fields

Nevertheless, from nucleon-nucleon scattering experiments we know that several additional meson fields are important and must be included for a quantitative description:[9]

- Isovector pseudoscalar (π^+, π^-, π^0)
- Isovector vector (ρ^+, ρ^-, ρ^0)
- Neutral pseudoscalar (η)

The next step in the systematic development of the theory should be the inclusion of these additional meson fields.

Fig. 2: Effective charge density for a point electron. The intrinsic charge density is the positive delta function indicated by the vertical dashed line at the origin.

5. Quantum Spreading Function

Although effects due to the intrinsic size of the nucleon dominate those due to quantum uncertainty, it is nevertheless important to provide an estimate of the effects of quantum uncertainty and to include them in the equations of motion if they are important. This should be possible by use of techniques analogous to those used by Moniz and Sharp[10] for nonrelativistic quantum electrodynamics.

5.1. Nonrelativistic Quantum Electrodynamics

There appear in the classical nonrelativistic equations of motion for an extended electron terms of the form $\int_\infty \int_\infty \rho(r)\, \mathcal{O} \rho(r')\, d^3r\, d^3r'$, where the operator $\mathcal{O}$ is a function of $\mathbf{r}$ and $\mathbf{r'}$. Moniz and Sharp[10] have shown in nonrelativistic quantum electrodynamics that the effect of quantum mechanics on the equations of motion is to replace such terms by terms of the form $\int_\infty \int_\infty \rho(r)\, \mathcal{O}_{\text{eff}}\, \rho_{\text{eff}}(r')\, d^3r\, d^3r'$ and derivatives with respect to λ of these terms. The effective operator $\mathcal{O}_{\text{eff}}$ is a function of $\mathbf{r}$, $\mathbf{r'}$, and $\lambda^2\, \nabla_{\mathbf{r'}}{}^{\mathbf{2}}$. The effective density ρ_{eff} is given by

$$\rho_{\text{eff}}(r) = \int_\infty S(|\mathbf{r} - \mathbf{r'}|)\, \rho(r')\, d^3r' \ , \tag{3}$$

where

$$S(|\mathbf{r} - \mathbf{r'}|) = -\frac{\cos(2|\mathbf{r} - \mathbf{r'}|/\lambda)}{\pi \lambda^2\, |\mathbf{r} - \mathbf{r'}|} \tag{4}$$

Fig. 3: Effective mass density for an extended nucleon. The intrinsic mass density is exponential with a root-mean-square radius of 0.862 fm.

and $\lambda = 1/(M_0)$ is the Compton wavelength associated with the particle's bare mass. [Please note that we have made some obvious corrections to eq. (3.27) of ref. 10.]

For a point electron, where the intrinsic charge density is $\rho_{\mathrm{e}}(r) = \delta_+(r)/(4\pi r^2)$, eq. (3) reduces to

$$\rho_{\mathrm{eff}}(r) = -\frac{\cos(2r/\lambda_{\mathrm{e}})}{\pi {\lambda_{\mathrm{e}}}^2 \, r} \ , \tag{5}$$

with a Compton wavelength $\lambda_{\mathrm{e}} = 386.15933$ fm. As illustrated in fig. 2, the effective charge density for a point electron is a decreasing oscillatory function of radial distance from the origin.

5.2. Classical Hadrodynamics

For an extended nucleon, where the intrinsic mass density is given by the exponential (1), eq. (3) leads to

$$\rho_{\mathrm{eff}}(r) = \frac{\mu^3}{8\pi\lambda\, r} \int_0^\infty r' \exp(-\mu r') \left\{ \sin(2|r-r'|/\lambda) - \sin[2(r+r')/\lambda] \right\} dr' \ . \tag{6}$$

For a bare nucleon mass[4] $M_0 = 949.47$ MeV, the Compton wavelength $\lambda = 0.20783$ fm, which is much smaller than the root-mean-square radius. This has the consequence, as illustrated in fig. 3, that the effective mass density for an extended nucleon oscillates around the intrinsic mass density as a function of radial distance from the origin.

6. Conclusions

To conclude, we have shown that classical hadrodynamics

- Provides a manifestly Lorentz-covariant microscopic many-body approach to relativistic heavy-ion collisions
- Satisfies *a priori* the basic conditions that are present
- Requires minimal physical input
- Leads to equations of motion that can be solved numerically without further approximation
- Contains an inherent spacetime nonlocality that may be responsible for significant collective effects
- Provides in its present form a qualitative description of transverse momentum, radiated energy, and other gross features in nucleon-nucleon collisions
- Should be extended to include additional meson fields for the π, ρ, and η, as well as quantum size effects

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